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Prize 2020

Joint modeling of the physical and pricing density

with applications

Master’s thesis in Mathematics

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KU LEUVEN

FACULTY OF SCIENCE

September 24, 2020

Outline

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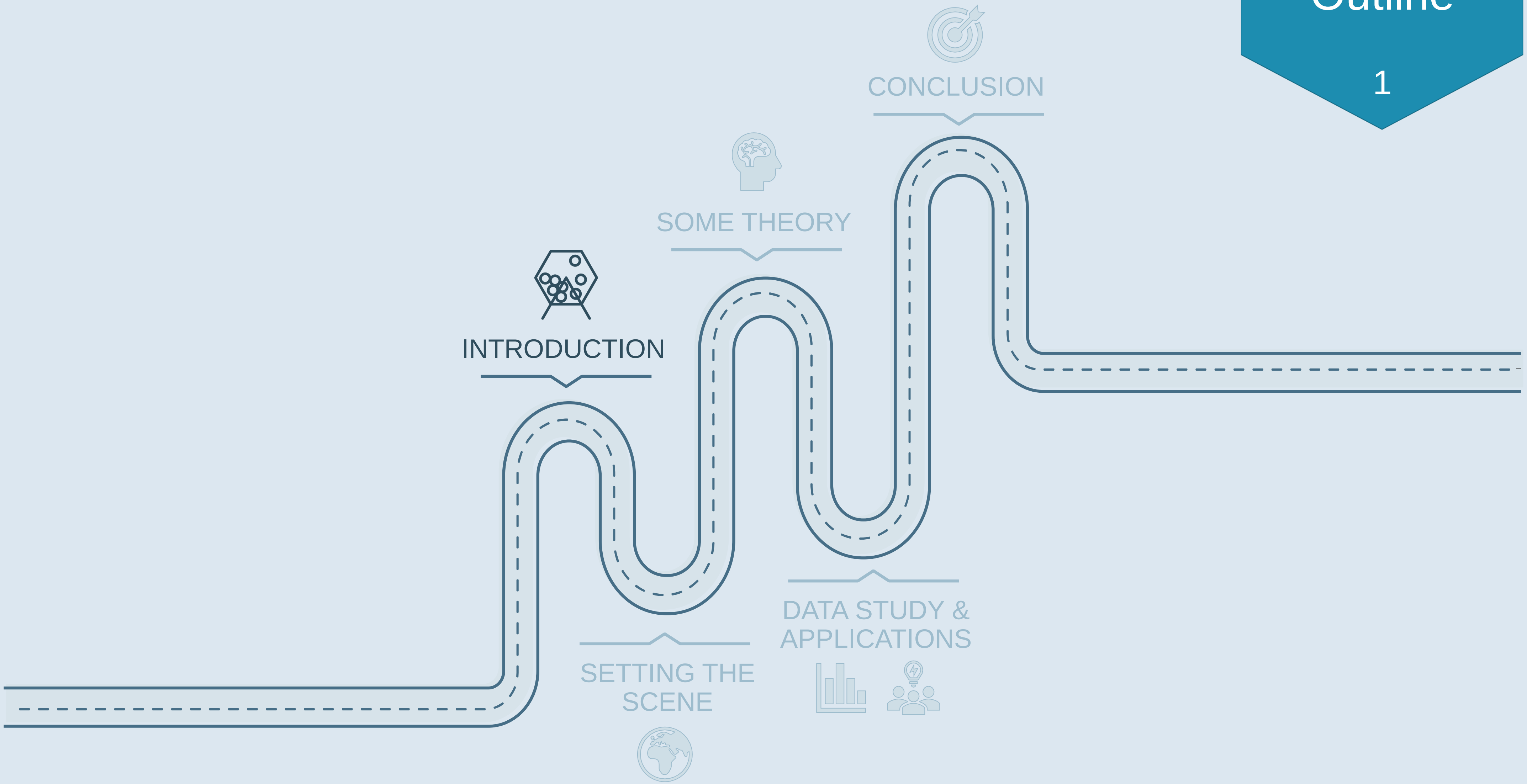
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Lotto

	Correct Numbers	Average Prize (€)	Odds
Rank 1	6	1 806 159.45	1 in 8 145 060
Rank 2	5 + bonus	59 878.78	1 in 1 357 510
Rank 3	5	1 423.85	1 in 35 724
Rank 4	4 + bonus	277.11	1 in 14 290
Rank 5	4	25.94	1 in 772
Rank 6	3 + bonus	10.48	1 in 579
Rank 7	3	6.25	1 in 48
Rank 8	2 + bonus	3.75	1 in 64
Rank 9	1 + bonus	1.25	1 in 18

Data retrieved from https://www.nationale-loterij.be/onze-spelen/lotto/statistieken

Lotto

Bet

€ 1.25

Average Revenue

€ 0.63

Average number of players / draw | 577 216

Lotto

Bet

€ 1.25

Financial Instrument

Price

€ X

Lotto

Financial Instrument

Lotto		Financial Instrument
Bet: € 1.25	↔	**Price**: € X
Average Revenue: € 0.63	↔	**Average Payout**: € Y

Lotto

Bet
€ 1.25

<

Average Revenue
€ 0.63

Financial Instrument

Price
€ X

<

Average Payout
€ Y

X > Y

Why are people willing to pay more?

For which instruments?

How to determine the average payout?

Outline

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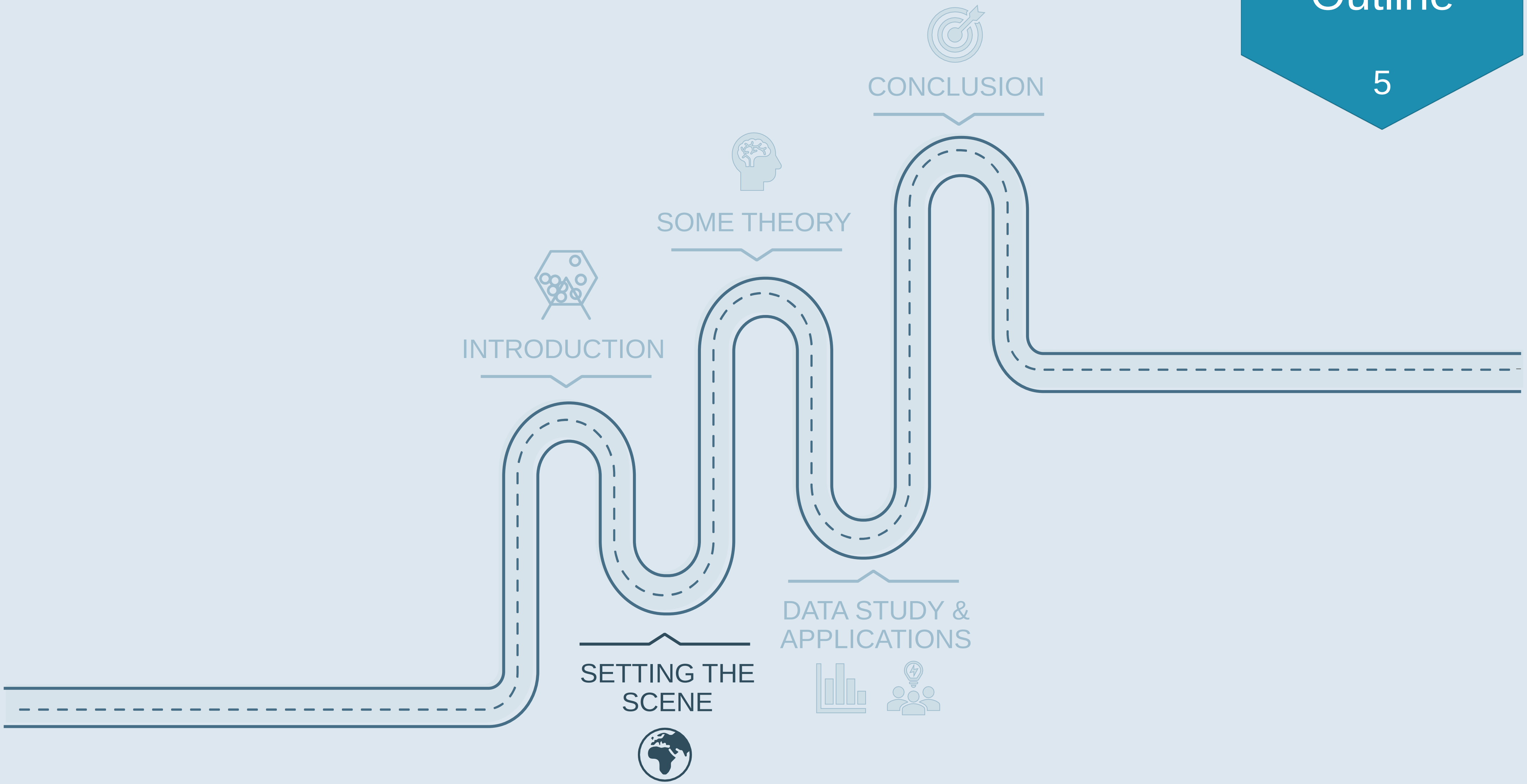

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P-world

› Physical world in which payoffs are realized

› Physical density p used to calculate expected payoffs $= \int \text{payoff} \cdot p$

Expected payoff = € 0.63 = E_P(payoff Lotto)

Q-world

› Artificial setting under which one determines prices

› Pricing density q used to estimate prices $\approx \int \text{payoff} \cdot q$

Price ticket = € 1.25 $\approx E_Q$(payoff Lotto)

This master's thesis focusses on the interplay between P and Q.

To build a bridge…

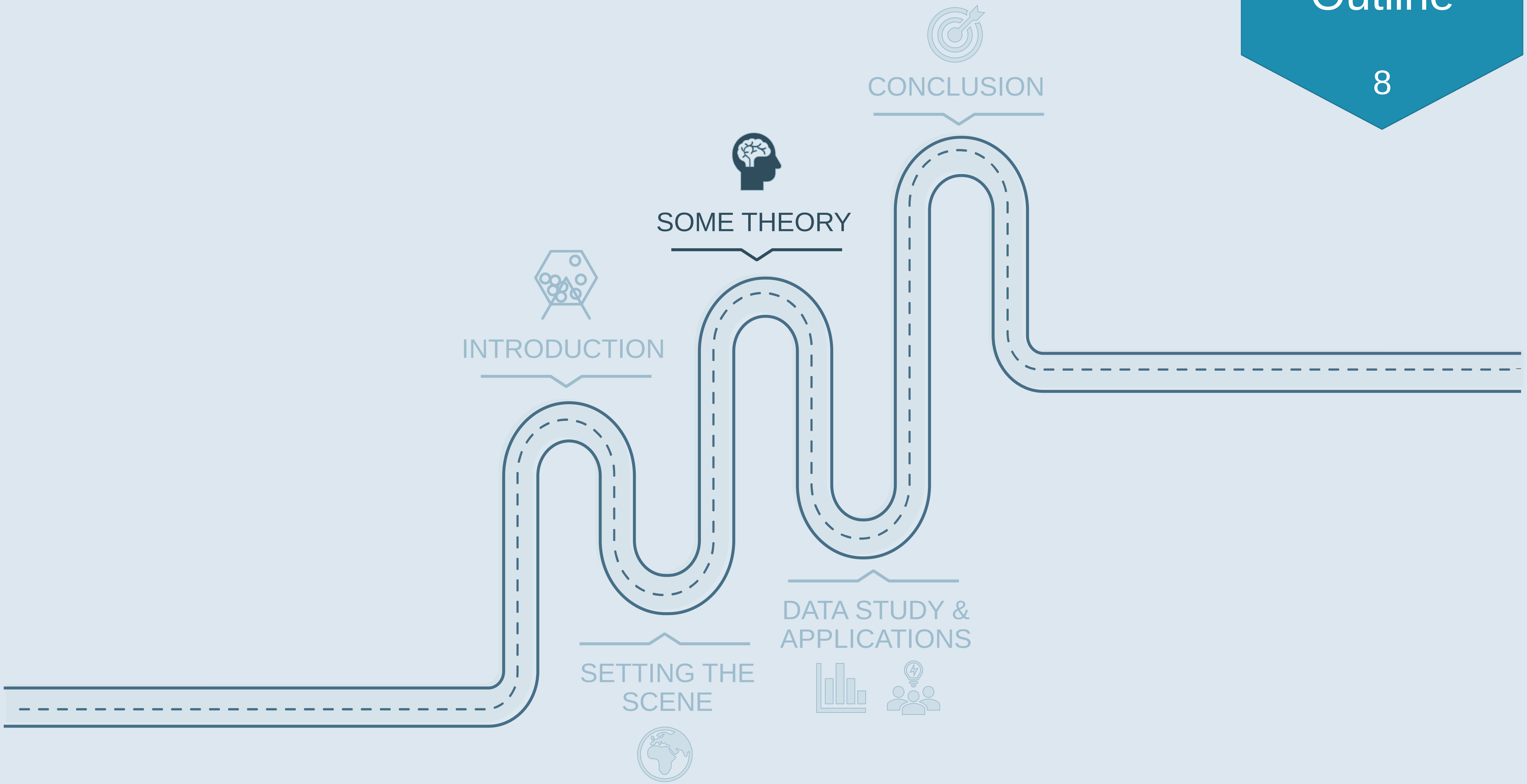
This master's thesis focusses on the interplay between P and Q.

To build a bridge…

…you need bricks.

- **Physical density $\boldsymbol{p}$** in order to calculate $E_P(\text{payoff}) = \int \text{payoff} \cdot p$
- **Pricing density $\boldsymbol{q}$** in order to calculate $E_Q(\text{payoff}) = \int \text{payoff} \cdot q$

Outline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Estimation methods

Physical density p

Historical data on the return of the asset

- Backward looking
- One new observation each day

Pricing density q

Option data and pricing model

- Forward looking
- Multiple new observations each day

Estimation methods

Physical density p

Historical data on the return of the asset

- Backward looking
- One new observation each day

Pricing density q

Option data and pricing model

- Forward looking
- Multiple new observations each day

Is there a way to also estimate physical densities from option data?

History of pricing models

1973
Black-Scholes-Merton model

1997
Nobel prize for Merton and Scholes

1998
General Variance Gamma model

2007
Bilateral Gamma model

INCREASED ACCURACY

“All models are wrong,
but some are useful.”

- George E.P. Box

History of pricing models

1973
Black-Scholes-Merton model

1997
Nobel prize for Merton and Scholes

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Bilateral Gamma model

2019
We go one step further!

INCREASED ACCURACY

Tilted Bilateral Gamma model

- Extension of the Bilateral Gamma model
- Allows for the joint estimation of the physical and pricing density from option prices

(Madan, Schoutens & Wang, 2020)

Tilted Bilateral Gamma model

› Extension of the Bilateral Gamma model

› Allows for the joint estimation of the physical and pricing density from option prices

ASSUMPTIONS

› Investors are risk-averse

› Investors have heterogeneous beliefs

- Long positions are allowed
- Short positions are allowed

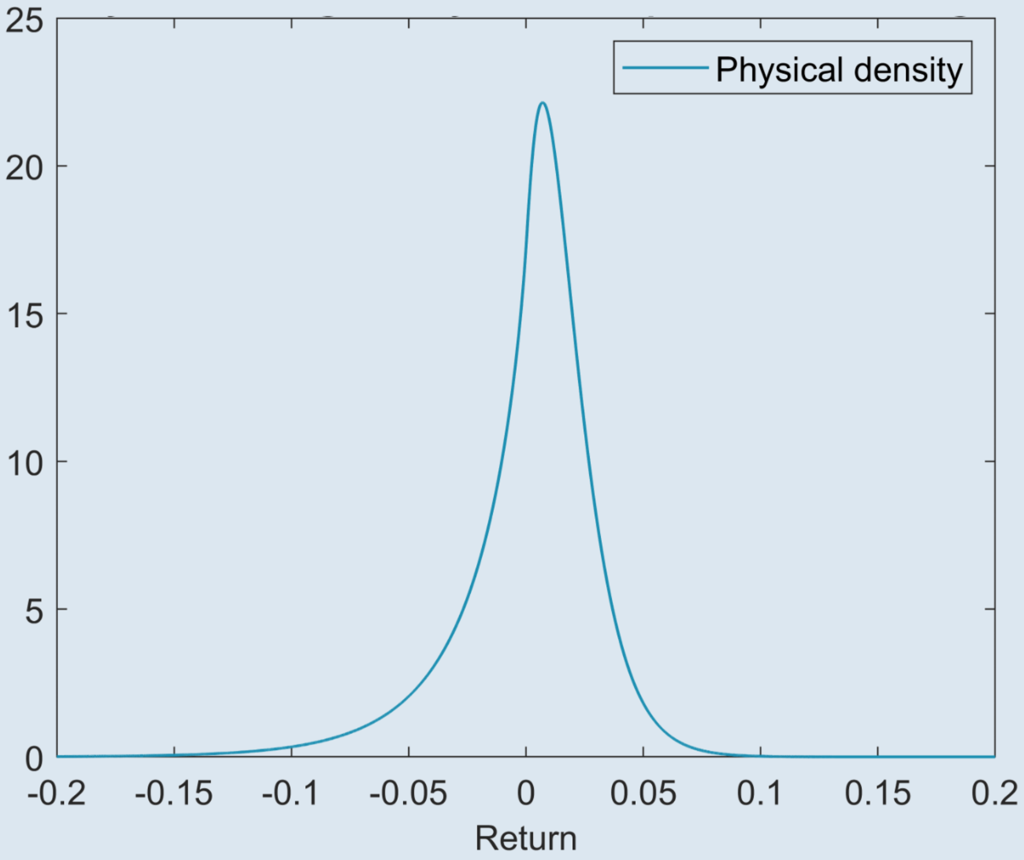
(Madan, Schoutens & Wang, 2020)

Tilted Bilateral Gamma model

Recall

- **Physical density**:
  what you *expect to get back* from the instrument

- **Pricing density:**
  what you are *willing to pay* for the instrument



Empirical evidence in e.g. (Bakshi et al., 2010)

Tilted Bilateral Gamma model

Recall

- **Physical density**:
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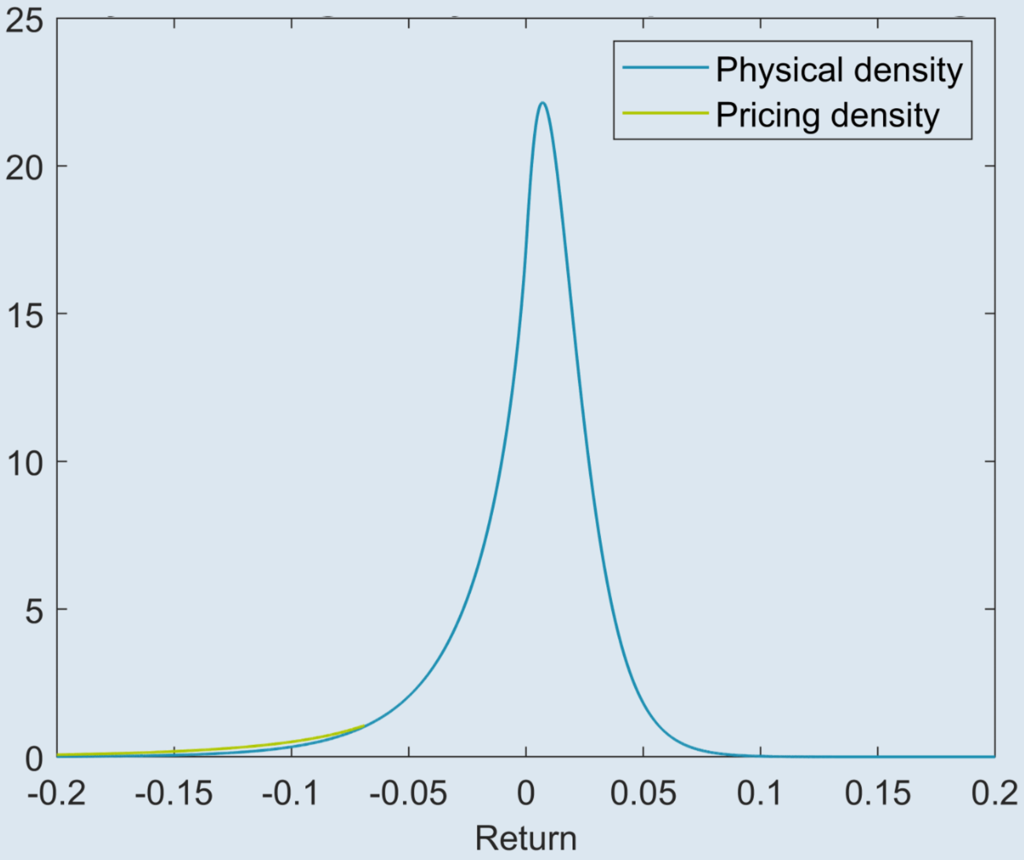
Risk-aversion and heterogeneous beliefs lead to

- **Long investor**:
  wealth loss in negative return state
  $\rightarrow$ loss protection leads to heavier left tail



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Risk-aversion and heterogeneous beliefs lead to

- **Long investor**:
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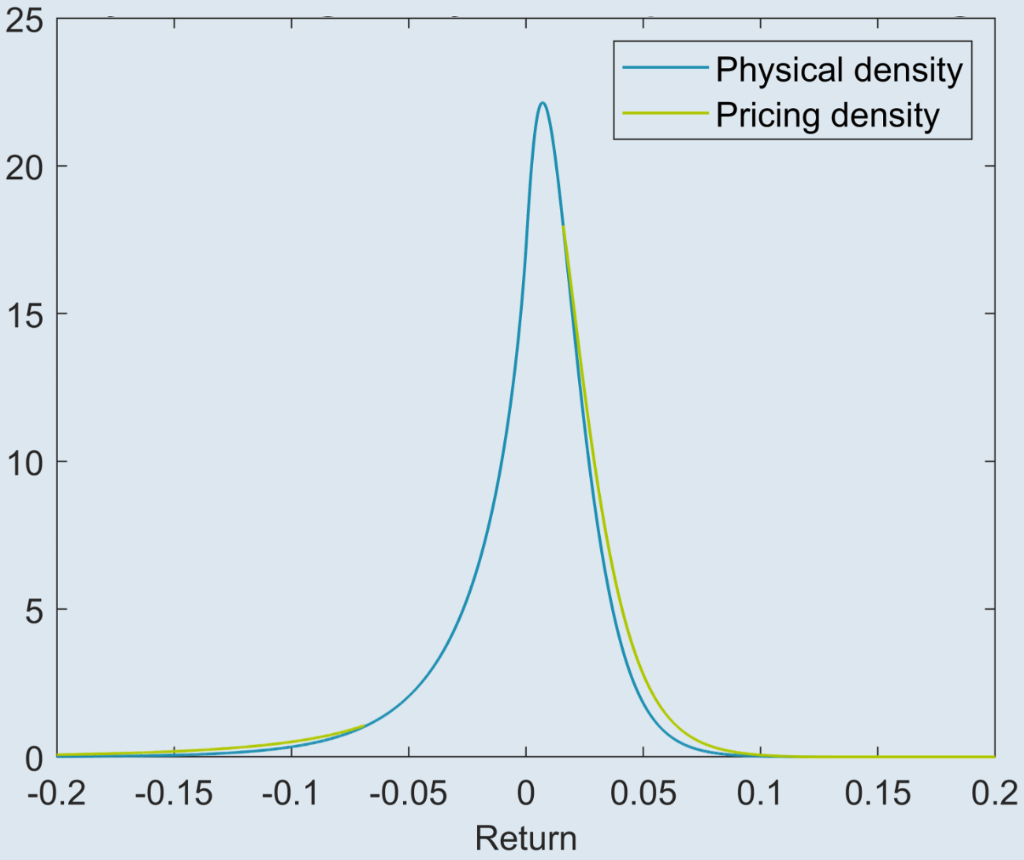
  $\rightarrow$ loss protection leads to heavier left tail
- **Short investor**:
  wealth loss in positive return state
  $\rightarrow$ loss protection leads to heavier right tail



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Tilted Bilateral Gamma model

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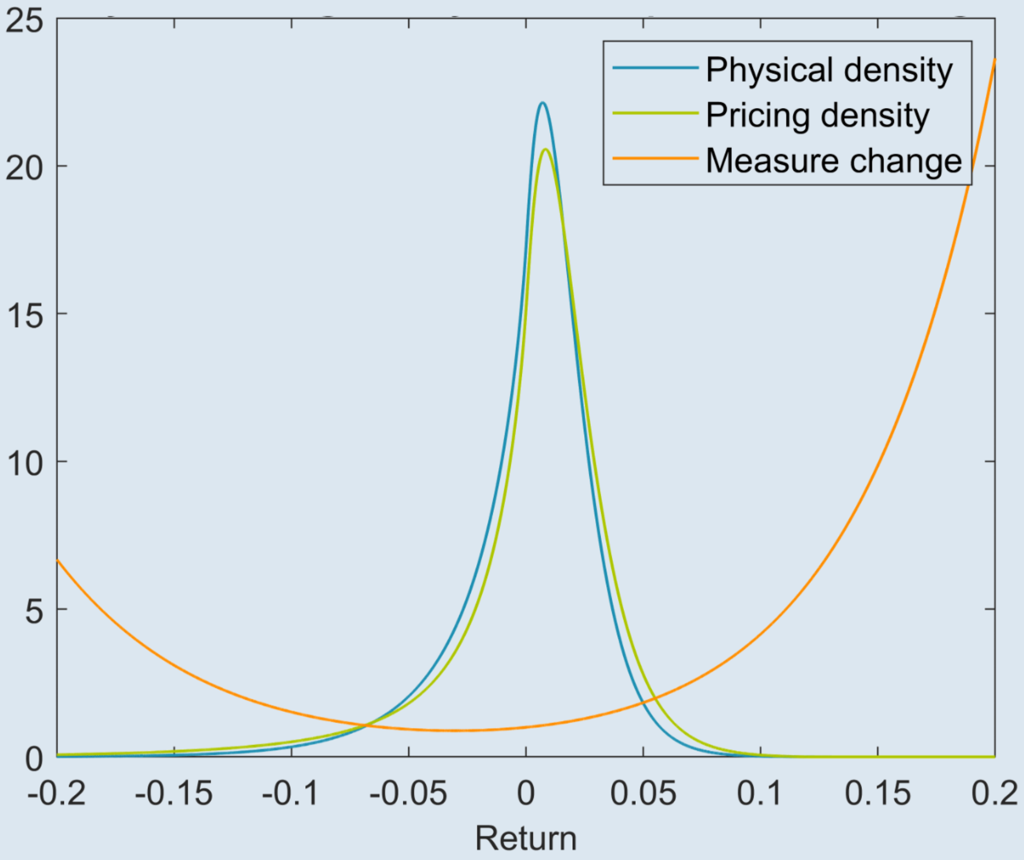


Empirical evidence in e.g. (Bakshi et al., 2010)

Tilted Bilateral Gamma model

Pricing density q = U-shape · physical density p

Tilted Bilateral Gamma model

Pricing density q = U-shape · physical density p

1

Physical density from Bilateral Gamma family

Tilted Bilateral Gamma model

Pricing density q = U-shape · physical density p

1 Physical density from Bilateral Gamma family

2 Pricing density follows a Tilted Bilateral Gamma model

Tilted Bilateral Gamma model

Pricing density q = U-shape · physical density p

1 Physical density from Bilateral Gamma family

2 Pricing density follows a Tilted Bilateral Gamma model

3 Pricing density can be estimated from option data

Tilted Bilateral Gamma model

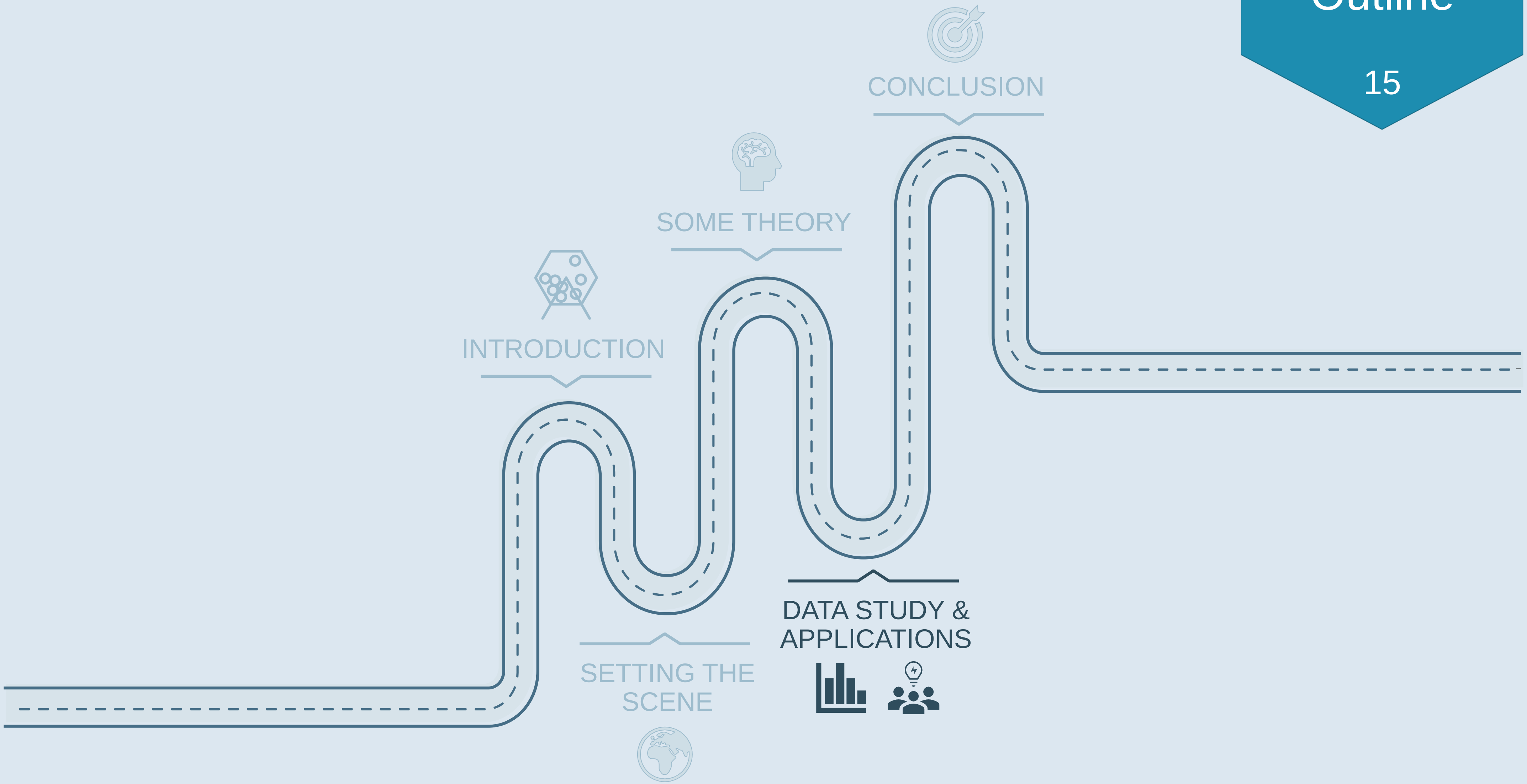
Pricing density q = U-shape · physical density p

1. Physical density from Bilateral Gamma family
2. Pricing density follows a Tilted Bilateral Gamma model
3. Pricing density can be estimated from option data
4. Physical density can be jointly estimated from option data

Tilted Bilateral Gamma model

Pricing density q = U-shape · physical density p

1. Physical density from Bilateral Gamma family
2. Pricing density follows a Tilted Bilateral Gamma model
3. Pricing density can be estimated from option data
4. Physical density can be jointly estimated from option data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Tilted Bilateral Gamma as option pricing model

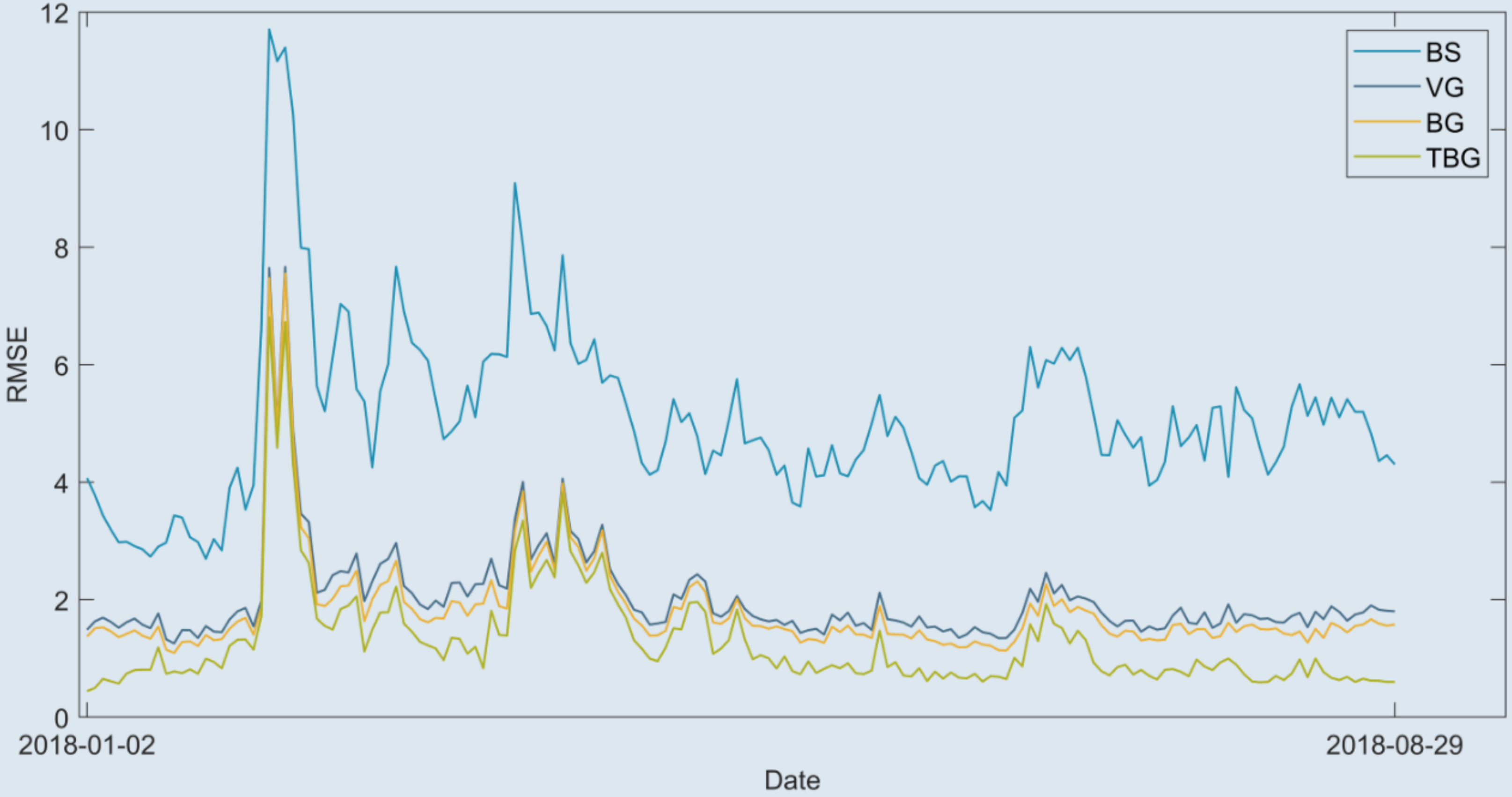


FIGURE Evolution of the RMSE over time between optimal Black-Scholes, Variance Gamma, Bilateral Gamma and Tilted Bilateral Gamma model prices and market prices of plain-vanilla options on the S&P500 index. A calibration is conducted on each business day between January 2, 2018 and August 29, 2018.

Master's thesis original application

Option positioning

Calibrate Tilted Bilateral Gamma model on option data S&P500

Physical density p

$\sim$ payout

Pricing density q

$\sim$ price

Option positioning

Calibrate Tilted Bilateral Gamma model on option data S&P500

Physical density p

$\sim$ payout

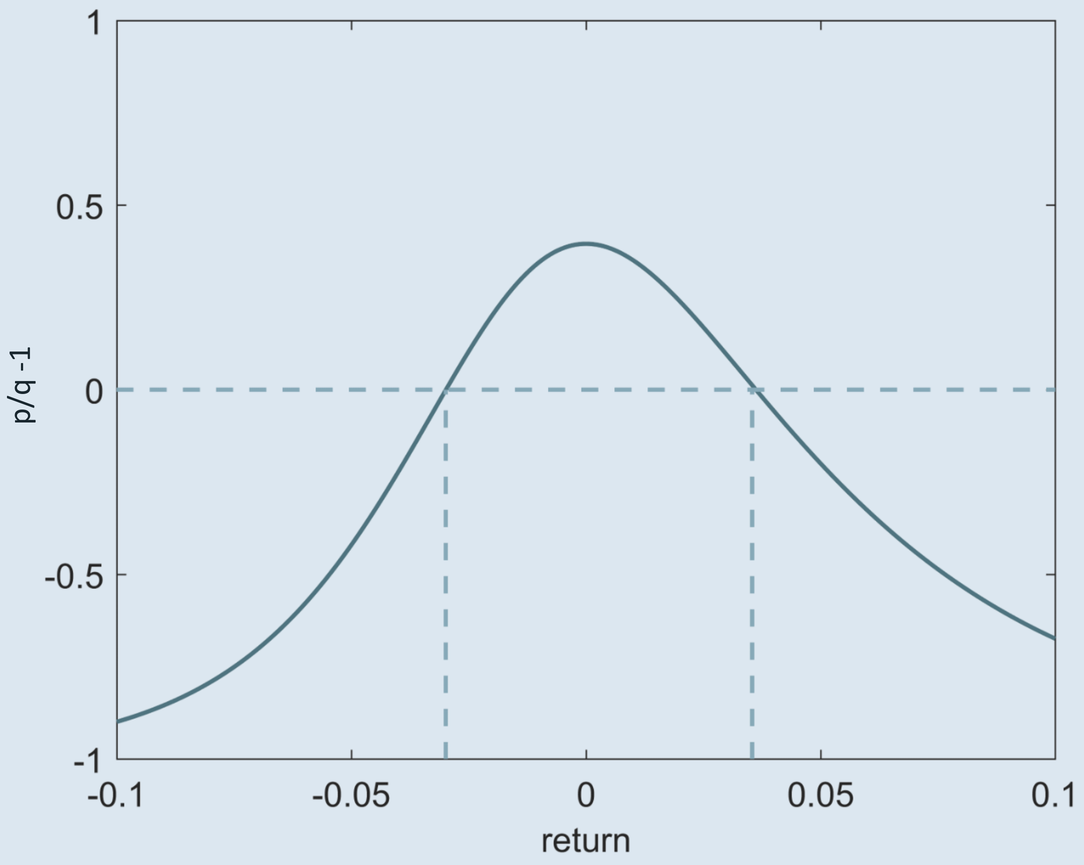
Pricing density q

$\sim$ price

Consider the curve $\frac{p-q}{q}$

$\sim$ expected return

Option positioning: the curve $\frac{p-q}{q}$



Option positioning

Calibrate Tilted Bilateral Gamma model on option data S&P500

Physical density p

~ payout

Pricing density q

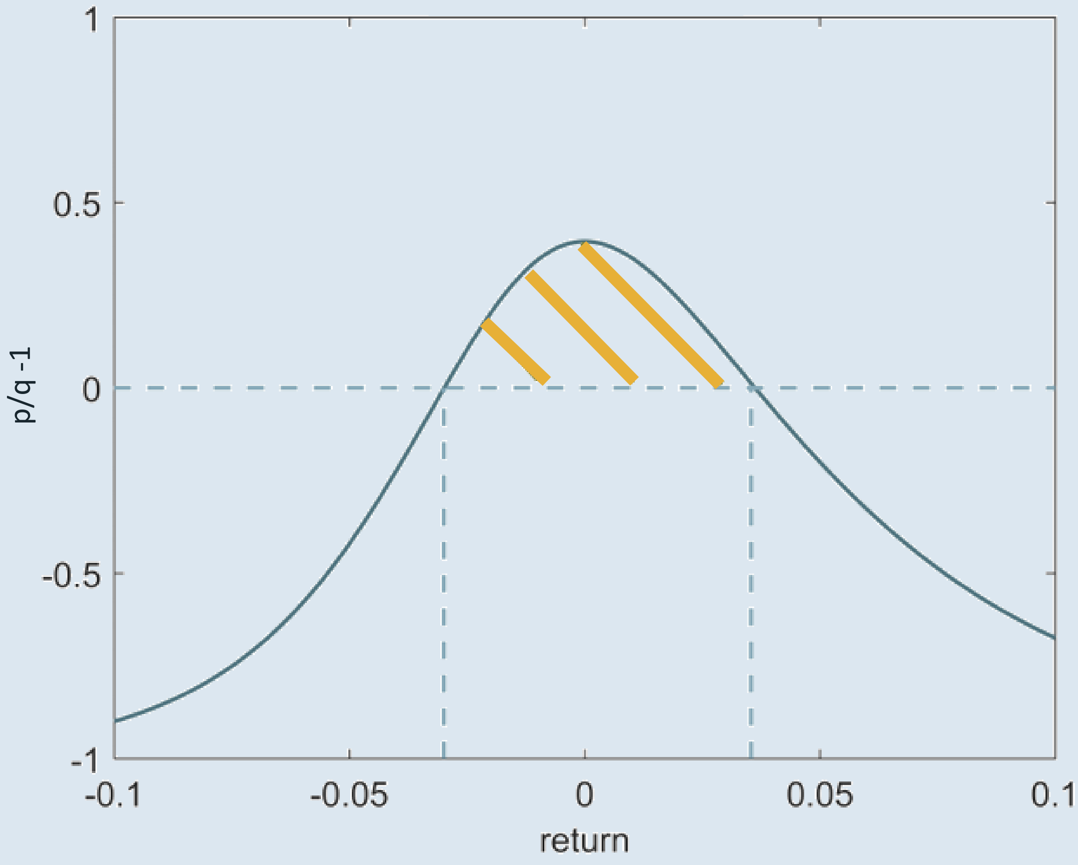
~ price

Consider the curve $\frac{p-q}{q}$

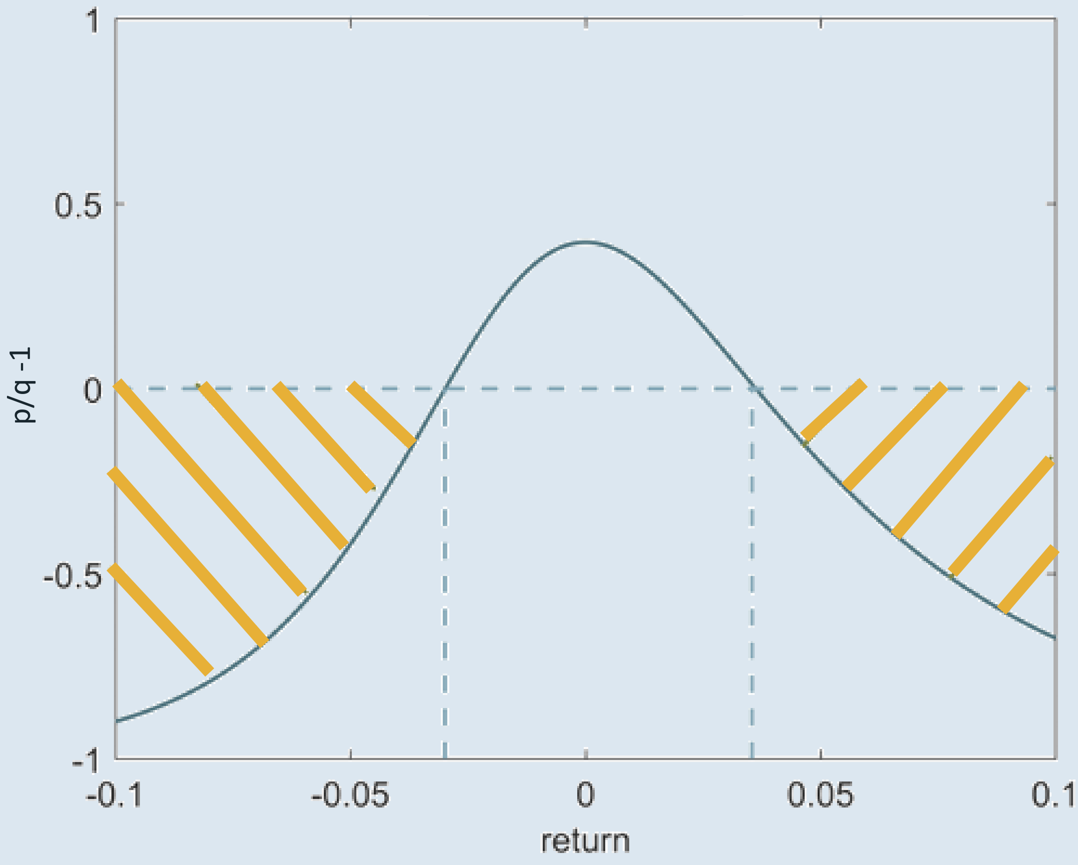
~ expected return

Price $\approx E_Q\left(\frac{p-q}{q}\right) = \int \left(\frac{p-q}{q}\right) \cdot q = 0$

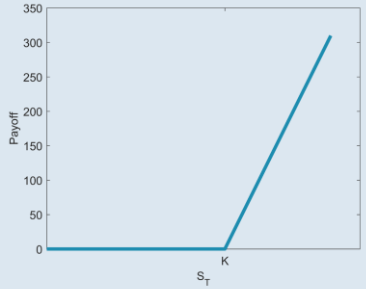
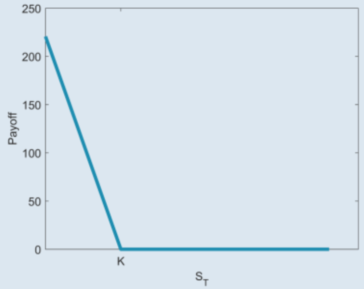
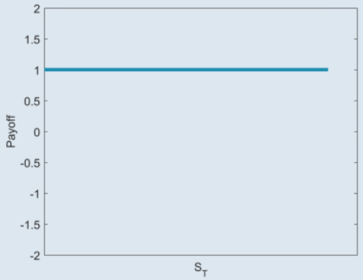
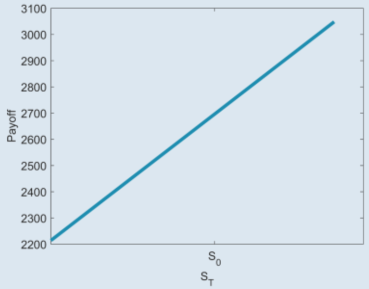
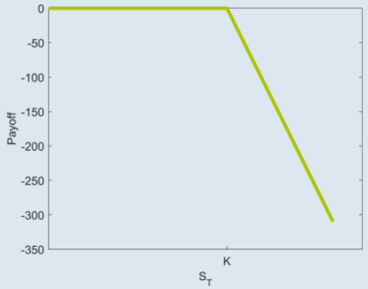
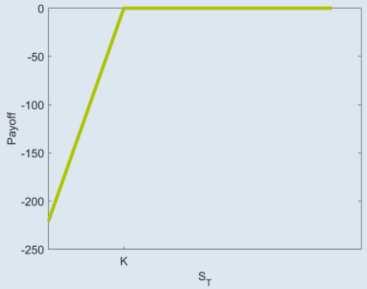
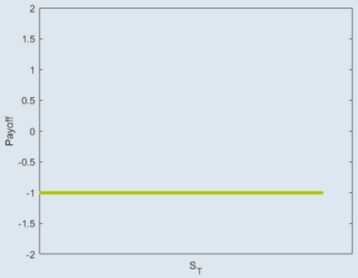
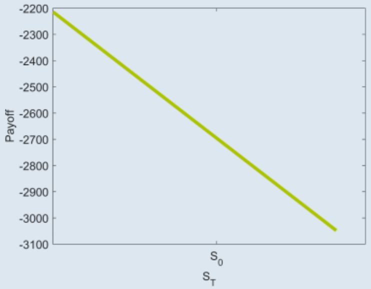
Option positioning: the curve $\frac{p-q}{q}$



Option positioning: the curve $\frac{p-q}{q}$



Option positioning: estimation of the curve

	Call option	Put option	Risk-free account	Stock
Buy				
Sell				

Option positioning strategy

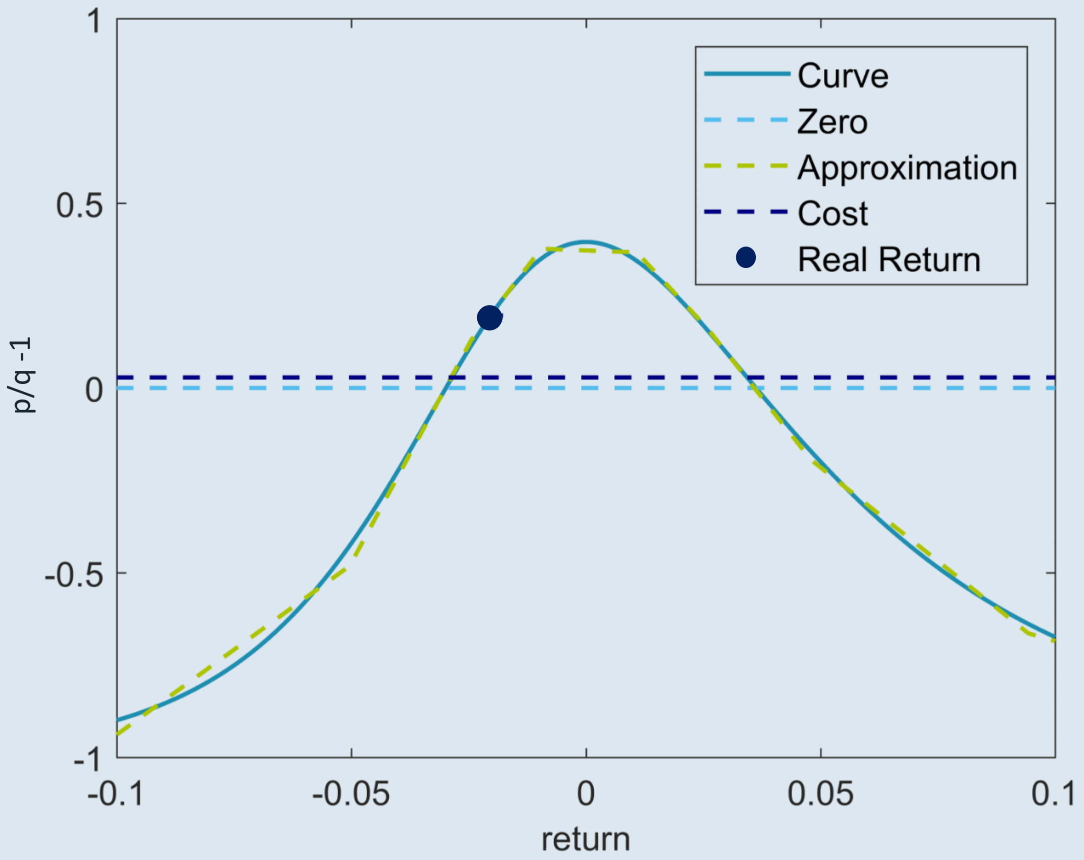
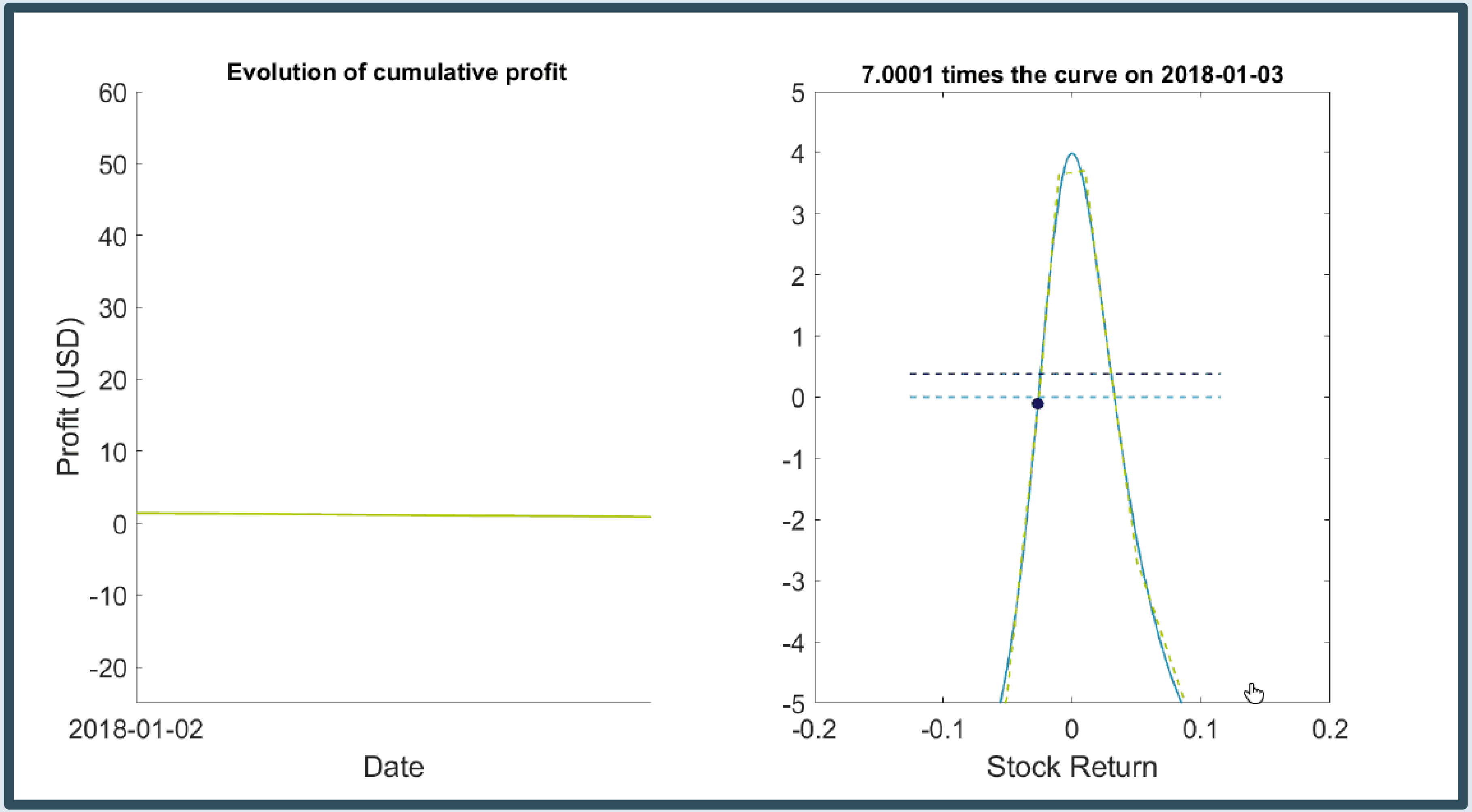


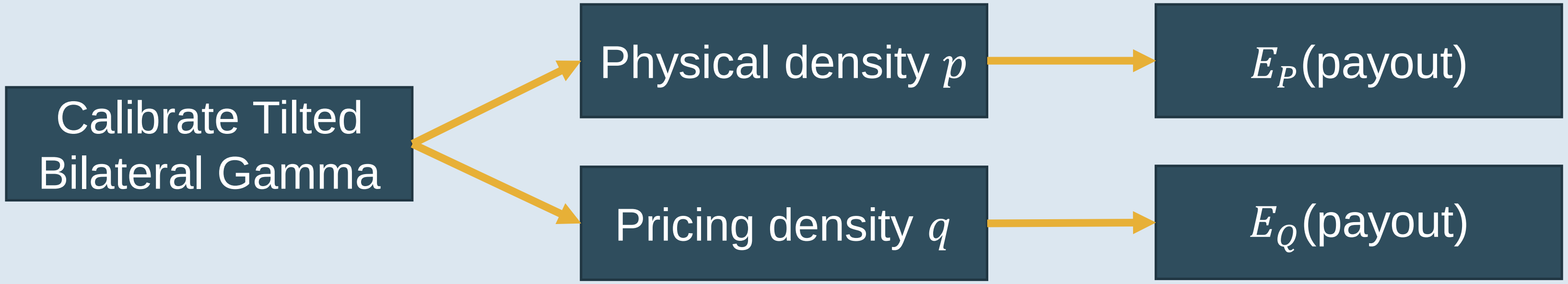
FIGURE Calculations are based on option data of the S&P500 index on January 2, 2018. The maturity is equal to 1 month.

Option positioning strategy



Additional application

The risk premium of a financial instrument



The risk premium of a financial instrument

Calibrate Tilted Bilateral Gamma

Physical density p → $E_P(\text{payout})$

Pricing density q → $E_Q(\text{payout})$

Risk Premium of a financial instrument

$$= \frac{E_P(\text{payout}) - E_Q(\text{payout})}{E_Q(\text{payout})}$$

The risk premium of a financial instrument

Calibrate Tilted Bilateral Gamma

Physical density p → $E_P(\text{payout})$

Pricing density q → $E_Q(\text{payout})$

Risk Premium of a financial instrument

$$= \frac{E_P(\text{payout}) - E_Q(\text{payout})}{E_Q(\text{payout})}$$

− Expensive instrument

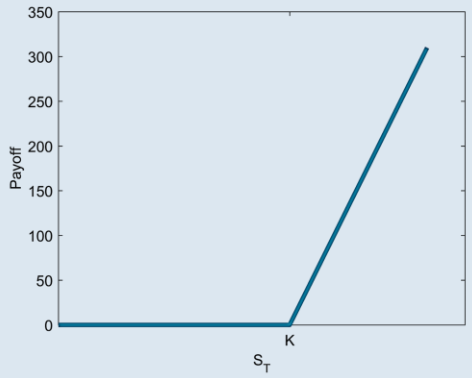
+ Cheap instrument

Zero-risk premium strike of a call option

- European Call option on asset S with strike K and maturity T

Payoff

$$= \begin{cases} S_T - K & K < S_T \\ 0 & K \geq S_T \end{cases}$$



Zero-risk premium strike of a call option

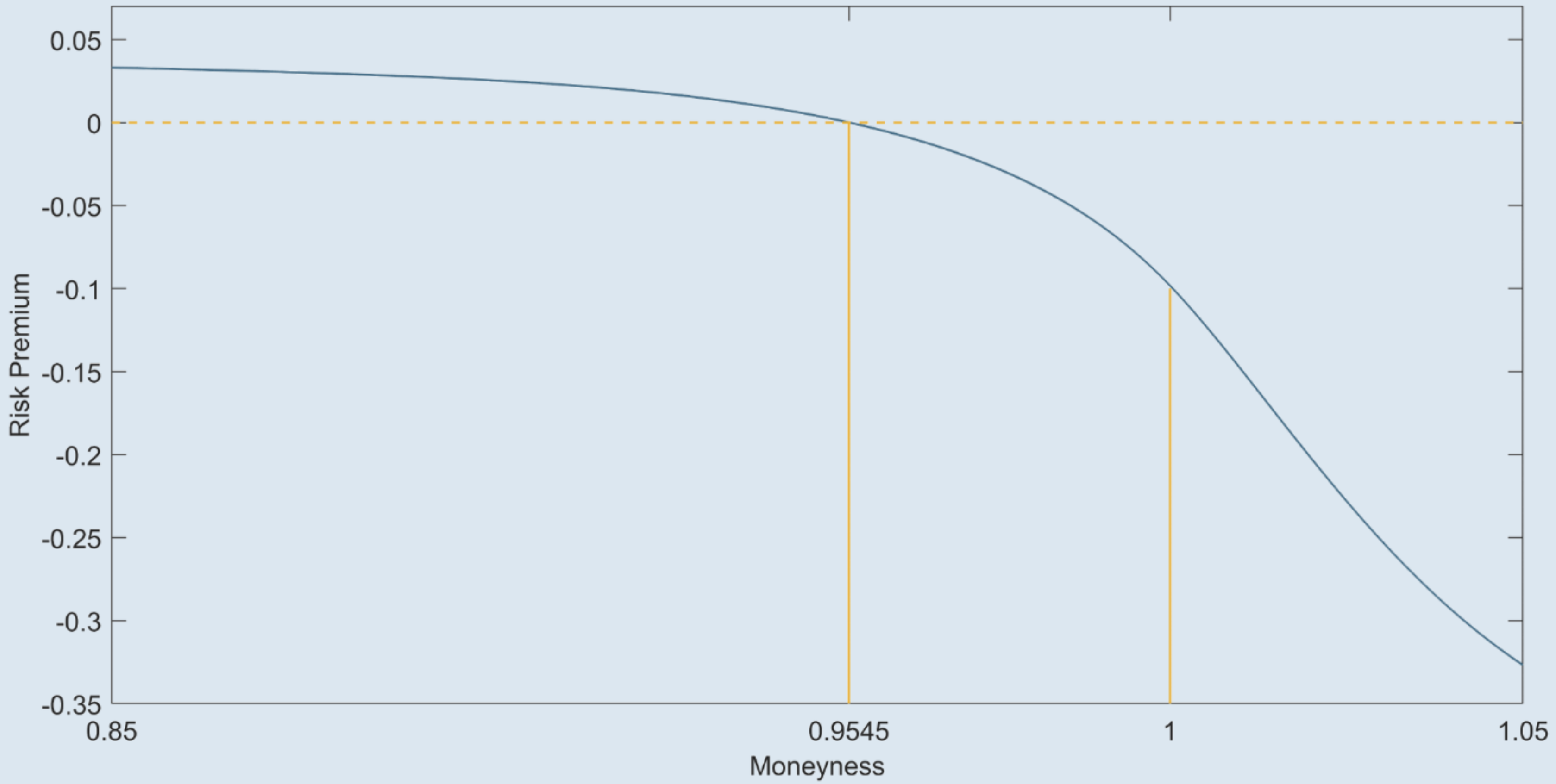


FIGURE Risk premium of the European call option with maturity T = 1 month and varying strikes with underlying the S&P500 index on March 15, 2018. The zero-risk premium moneyness level amounts around 95.45% of the spot price.

Zero-risk premium strike of a call option

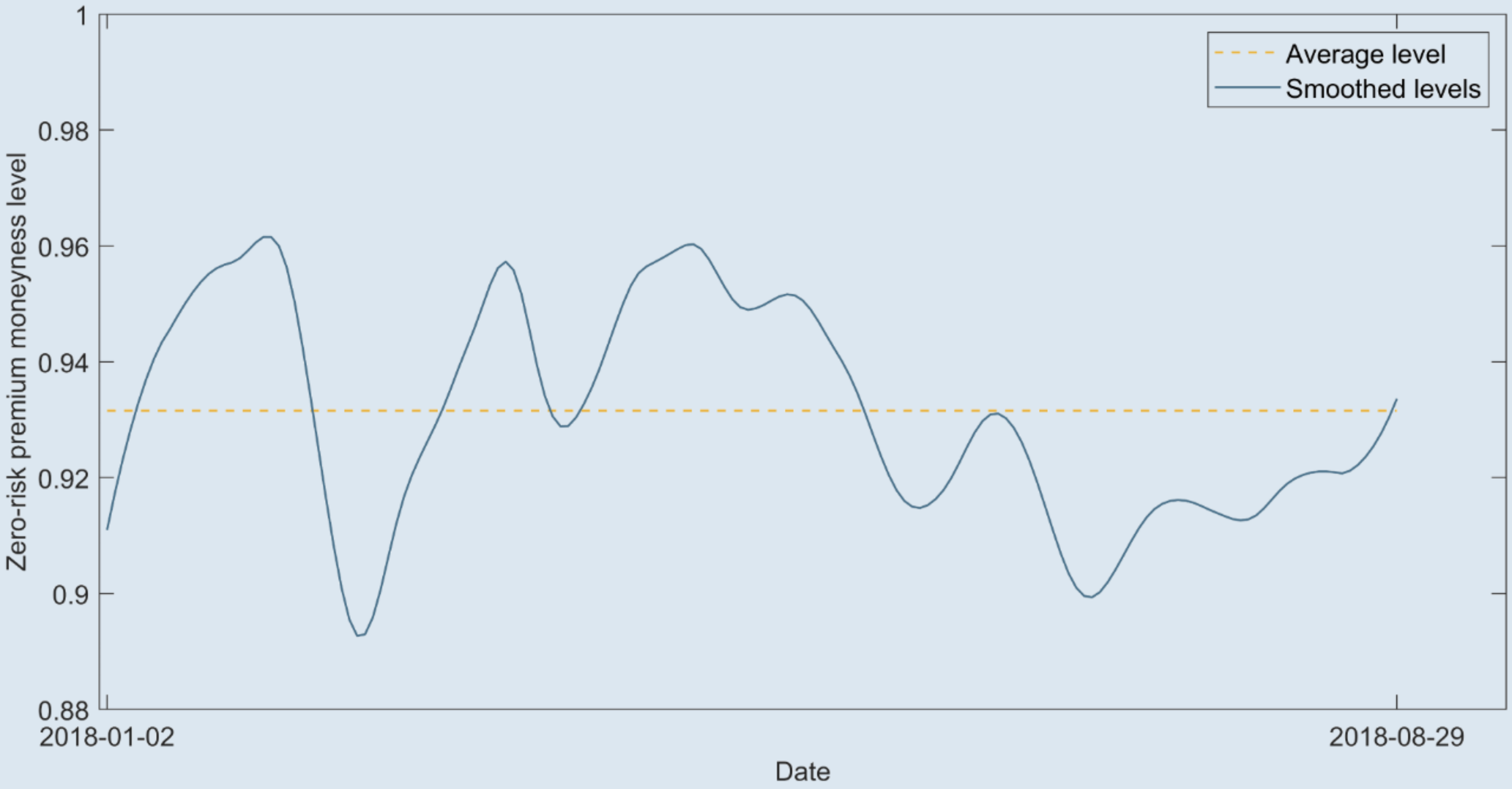


FIGURE Evolution over time of the zero-risk premium strike of a European call option on the S&P500 index, with a fixed maturity of 1 month. The average moneyness level amounts around 93.15%.

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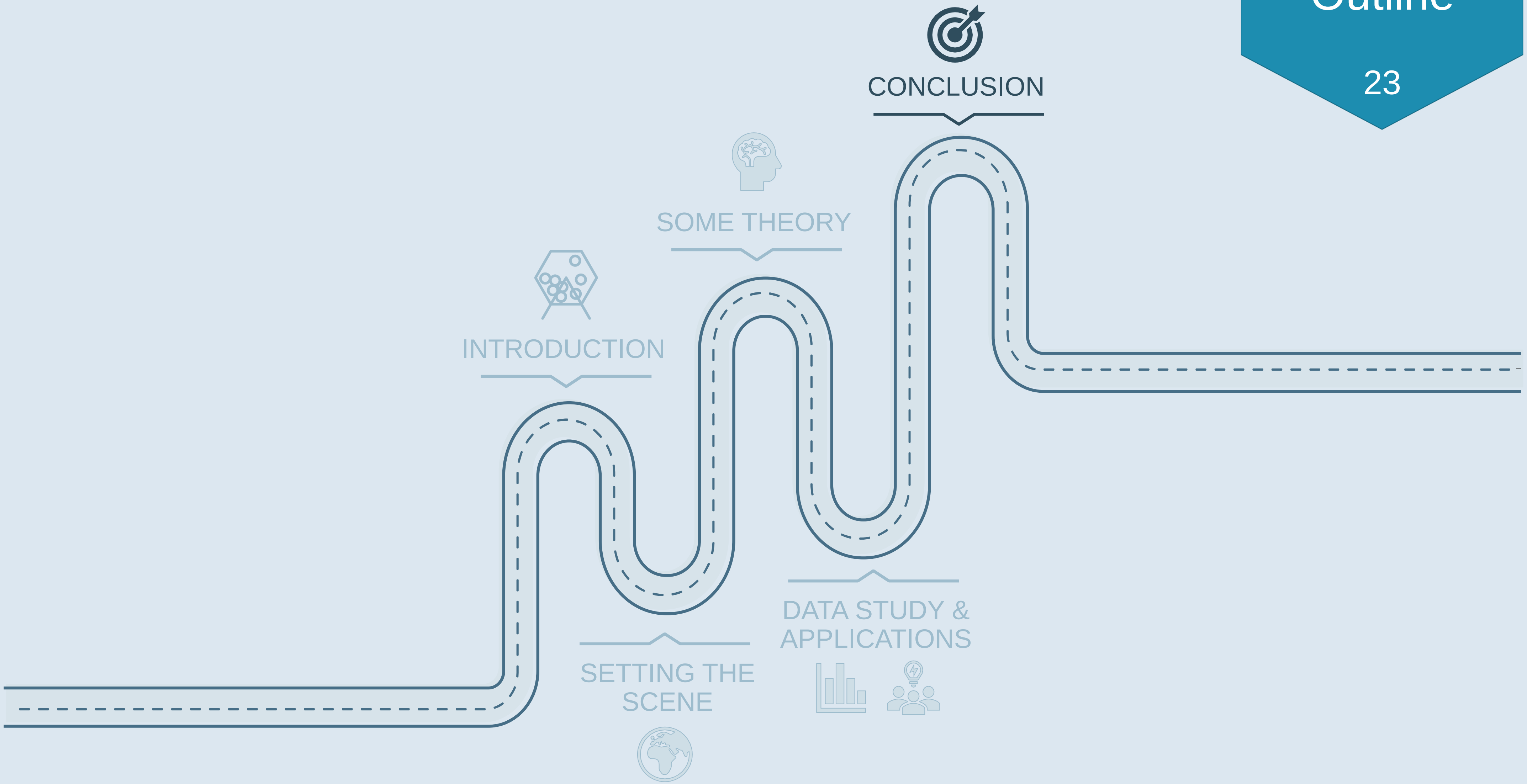
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Conclusion

Tilted Bilateral Gamma

- Allows for the simultaneous estimation of the physical and pricing density from option prices
- Outperforms classic option pricing models

Option positioning

- Set up an option positioning strategy with theoretical cost equal to 0
- Look at evolution of profit over time

Risk premium

- Risk premium of a call option decreases with moneyness
- Zero-risk premium strike call option on S&P 500 index situated in-the-money and rather stable over time

THANK YOU!

› Bakshi, G., Madan, D. B. and Panayotov, G. (2010)..*Returns of claims on the upside and the viability of U-shaped pricing kernels.* Journal of Financial Economics 97(1), 130-154.

› Madan, D. B., Schoutens, W. and Wang, K. (2020), *Bilateral multiple Gamma returns: their risks and rewards.* International Journal of Financial Engineering 7(1).

› Verschueren, E., Höcht, S., Madan, D. B., Schoutens, W. (2020). *It takes two to tango: estimation of the zero-risk premium strike of a call option via joint physical and pricing density modeling.* Manuscript submitted for publication.